Функции Грина

Теорема Вика

$$\langle 0|\varphi_1\varphi_2\varphi_3\varphi_4|0\rangle = \langle 0|\varphi_1\varphi_2|0\rangle\times\langle 0|\varphi_3\varphi_4|0\rangle + \langle 0|\varphi_1\varphi_3|0\rangle\times\langle 0|\varphi_2\varphi_4|0\rangle \pm \langle 0|\varphi_1\varphi_4|0\rangle\times\langle 0|\varphi_2\varphi_3|0\rangle$$

Fermi $\to \ominus$

$$\frac{N!}{2^{N/2}\left(\frac{N}{2}\right)!} = \frac{4!}{4\cdot 2!} = 3$$

$$E_p = \sqrt{p^2+m^2}$$

$$D^R(\omega,\vec{p}) = \int_0^\infty dt\, e^{i\omega t}\underline{\int d^3x\, e^{-i\vec{p}\vec{x}}}\cdot\left[D(t,\vec{x}) - D(-t,-\vec{x})\right] \ominus$$

$$= \qquad D(t,\vec{x}) = \int\frac{d^3\vec{q}}{2E_{\vec{q}}}\, e^{-iE_p t + i\vec{q}\vec{x}}$$

$$\int d^3x \ldots = (2\pi)^3\delta(\vec{p}-\vec{q})$$

$$\ominus \int_0^\infty dt\, e^{i\omega t}\cdot\frac{1}{2E_p}\left\{e^{-iE_p t} - e^{iE_p t}\right\} \ominus$$

$$\int_0^\infty e^{i\Omega t}dt = \lim_{\epsilon\to 0}\int_0^\infty e^{i\Omega t-\epsilon t}dt = \frac{1}{\epsilon - i\Omega} = \frac{i}{\Omega + i0} = i\left\{\mathcal{P}\frac{1}{\Omega} - i\pi\delta(\Omega)\right\}$$

$$\ominus \frac{1}{2E_p}\left\{\frac{i}{\omega - E_p + i0} - \frac{i}{\omega + E_p + i0}\right\} = \frac{i}{(\omega+i0)^2 - E_p^2} =$$

$$= \frac{i}{(\omega+i0)^2 - \vec{p}^2 - m^2} = \frac{i}{\underbrace{\omega^2 - \vec{p}^2}_{p=(\omega,\vec{p})} - m^2 + i0\cdot\operatorname{sign}\omega} = \frac{i}{p^2 - m^2 + i0} = D^R(p)$$

$$(p^2 - m^2)D^R(p) = i$$

$$\boxed{(-\partial^2 - m^2)D^R(x-y) = i\delta^{(4)}(x-y)} \qquad D^R(t<0) \equiv 0$$

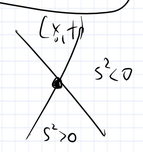
$$\partial_t^2 - \nabla^2$$

(?) $s^2 = (x^0 - y^0)^2 - (\vec{x}-\vec{y})^2 \lessgtr 0$

$D^R(s^2) \equiv 0 \qquad > 0$

$$D^R(t,\vec{x}) = \int (d^3\vec{p})\,\frac{1}{2E_p}(-2i)\sin E_p t\cdot e^{i\vec{p}\vec{x}} =$$

$$\| \qquad D^R(0,\sqrt{\vec{x}^2 - t^2}) = 0$$



Если $s^2 > 0$, s

$$D^R(t,\vec{x}) = D^R(\sqrt{t^2-x^2},0) = \boxed{\int\frac{(d^3p)}{2E_p}(-2i)\sin E_p s}$$

$$D^F(x-y) = \langle 0|\hat{T}\{\hat{\varphi}(x)\hat{\varphi}(y)\}|0\rangle =$$

$$= \begin{cases}\langle 0|\hat{\varphi}(x)\hat{\varphi}(y)|0\rangle, & x^0 > y^0\\ \langle 0|\hat{\varphi}(y)\hat{\varphi}(x)|0\rangle, & x^0 < y^0\end{cases}$$

$$\hat{U}(t,t') = \hat{T}\left\{\exp\left(-i\int_{t'}^{t}\hat{H}(\tau)d\tau\right)\right\} = \ldots + \frac{(-i)^2}{2!}\int_{t'}^{t}dt_1\int_{t'}^{t}dt_2\,\underbrace{\hat{T}\{\hat{\varphi}^4(x_1)\hat{\varphi}^4(x_2)\}}_{\langle 0\rangle = D^F(\ldots)D^F(\ldots)\ldots}$$

$$\int_{t'}^{t}\hat{\varphi}^4(\vec{x})dx$$

$$D^F(t,\vec{x}) = \underbrace{\Theta(t)D(t,\vec{x})} + \Theta(-t)D(-t,-\vec{x})$$

$$D^F(\omega,\vec{p}) = \underbrace{\int_0^\infty e^{i\omega t}dt\cdot\int d^3\vec{x}\, e^{-i\vec{p}\vec{x}}\int\frac{(d^3\vec{q})}{2E_q}e^{+i\vec{q}\vec{x} - iE_q t}}_{(2\pi)^3\delta(\vec{p}-\vec{q})} + \int_{-\infty}^{0}e^{-i\vec{q}\vec{x} + iE_q t}$$

$$= \int_0^\infty\frac{dt}{2E_p}e^{i(\omega - E_p)t} + \int_{-\infty}^{0}e^{i(\omega + E_p)t}\frac{dt}{2E_p} =$$

$$\lim_{\epsilon\to 0}\int e^{\epsilon t}\ldots = \frac{1}{\epsilon + i(\omega + E_p)} = \frac{-i}{\omega + E_p - i0}$$

$$= \frac{1}{2E_p}\left\{\frac{i}{\omega - E_p + i0} - \frac{i}{\omega + E_p - i0}\right\} = \frac{i}{\omega^2 - E_p^2 + i0} = \boxed{\frac{i}{p^2 - m^2 + i0}}$$

$$\omega^2 = E_p^2 - i\epsilon$$

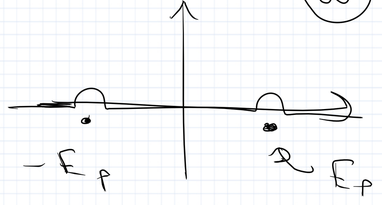
$$\omega = \pm(E_p - i0)$$

$$\pm E_p(1 - i\tilde{\epsilon})$$

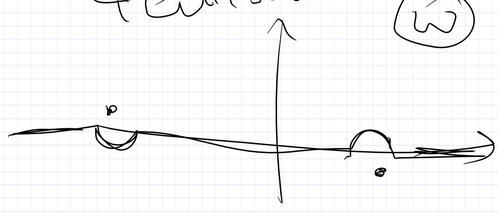
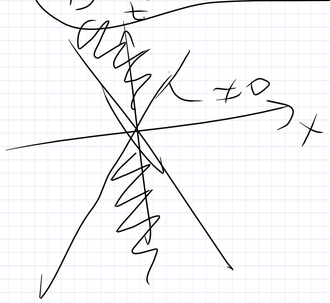
Запаздыв. ω

$$\text{у } D^R(\omega) = \frac{1}{(\omega+i0)^2 - E_p^2}$$

$$\omega = \pm E_p - i0$$



Фейнман ω



$$D^F(s^2) \neq 0 \qquad s^2 > 0,\ s^2 < 0$$