К.Г.

$$S = \int d^4x \left[(\partial_\mu \phi)^2 - m^2\phi^2 \right] \tfrac{1}{2}$$

$$\mathcal{L} = \left((\partial_t \phi)^2 - (\nabla\phi)^2 - m^2\phi^2 \right)$$

$$\pi = \frac{\partial \mathcal{L}}{\partial(\partial_t \phi)} = \partial_t \phi \Rightarrow \mathcal{H} = \frac{1}{2}\pi^2 + \frac{1}{2}(\nabla\phi)^2 + \frac{m^2}{2}\phi^2$$

$$\frac{d\pi}{dt} = -\frac{\partial H}{\partial \phi} + \nabla \frac{\partial H}{\partial \nabla \phi} = -m\phi + \nabla^2\phi \Rightarrow (\partial_t^2 - \nabla^2)\phi = -m\phi, \quad \Box\phi = -m\phi$$

$$\frac{d\phi}{dt} = \pi \qquad \frac{d\hat{A}}{dt} = \frac{\partial \hat{A}}{\partial t} + i[\hat{H}, \hat{A}]$$

$$\partial_t q = -\mathrm{div}\, j \Rightarrow \partial_t q + \mathrm{div}\, j = 0, \quad \partial_\mu j^\mu = 0, \quad j^\mu = (q, \vec{j})$$

$$M \to KM, \quad T\Pi \to KT\Pi$$

$$L = \sum T - \sum V \qquad p_i = \frac{\partial L}{\partial \dot{q}_i} \xrightarrow{H} q_i(p_i)$$

$$\frac{\delta S}{\delta q_i} = 0 \Rightarrow \frac{\partial L}{\partial q_i} - \frac{d}{dt}\frac{\partial L}{\partial \dot q_i} = 0$$

$$H = \sum p_i \dot q_i - L \Big|_{p_i}^{q_i}$$

$$\frac{dp_i}{dt} = -\frac{\partial H}{\partial q_i}, \quad \frac{dq_i}{dt} = \frac{\partial H}{\partial p_i}$$

$dh \ll \lambda$; $\mathrm{div}\, \vec v = 0$, $\mathrm{rot}\, \vec v = 0$

$$\frac{dA}{dt} = \frac{\partial A}{\partial t} + \frac{\partial A}{\partial p}\frac{dp}{dt} + \frac{\partial A}{\partial q}\frac{dq}{dt} = \frac{\partial A}{\partial t} + \{H, A\}$$

$$\{B, A\} = \frac{\partial B}{\partial q}\frac{\partial A}{\partial p} - \frac{\partial A}{\partial q}\frac{\partial B}{\partial p}$$

$$\boxed{\{q_i, p_j\} = \delta_{ij}} \qquad q \to \hat q, \; p \to \hat p$$

$$[\hat x_i, \hat p_j] = i\hbar \delta_{ij} \qquad \hbar i \{\;\} \to [\;]$$

Нормальн моды

$\lambda \gg a \Leftrightarrow ka \ll 1$

$$h(\vec x) \leftarrow h_{ij} \Rightarrow h(\vec x)$$

$$\sum \left(\frac{\dot q_k^2 m}{2} + \frac{m q_k^2 \omega_k^2}{2} \right)$$

$$a_i \to \vec x$$

$$L = \sum_i L_i = \int d^3r \, \mathcal{L}(h, \dot h)$$

$$\phi^2 + \phi^4 + \phi^3 \qquad \sin\phi, \dot\phi^2$$

$$L = \int dr\, dr'\, h(r) K_{rr'} h(r'), \quad K_{rr'} = \delta(r - r')$$

$$\pi = \frac{\delta L}{\delta \dot h} = \frac{\partial \mathcal{L}}{\partial \dot h} \Rightarrow H = \int d^3r \, \pi \dot h - L, \quad L = \int d^3r\, \mathcal{L}$$

$$\frac{dh}{dt} = \frac{\delta H}{\delta \pi} = \frac{\partial \mathcal{H}}{\partial \pi} \qquad \boxed{\mathcal{H} = \pi \dot h - \mathcal{L}}$$

$$\frac{d\pi}{dt} = -\frac{\delta H}{\delta h} = -\frac{\partial \mathcal{H}}{\partial h} + \nabla \frac{\partial \mathcal{H}}{\partial (\nabla h)} \qquad \sum_i \frac{\partial}{\partial x_i}$$

$\nabla h \to h$, $\partial_t h \to \pi$

$$S \Rightarrow_{\pi, h} \hat{\mathcal{H}}(\hat\pi, \hat h)$$

$$[\hat h(y), \hat \pi(z)] = i\delta(y - z)$$

$$[\hat h(y), \hat h(z)] = [\hat \pi, \hat \pi] = 0$$

$$\{A, B\} = \int dx \left[\frac{\delta A}{\delta h(x)} \frac{\delta B}{\delta \pi(x)} - \dots \right]$$

$$\boxed{\delta(y - z) = \{h(y), \pi(z)\}} = \int dx \left[\frac{\delta h(y)}{\delta h(x)} \right] \cdot \frac{\delta \pi(z)}{\delta \pi(x)} = \int dx\, \delta(x - y)\, \delta(x - z) = \delta(y - z)$$

$$h(y) = \int \delta(x - y) h(x) dx$$

$$\phi + \alpha\Delta\phi \Leftarrow \Rightarrow \phi$$

$$S[\phi + \alpha\Delta\phi] = S[\phi + \alpha\Delta\phi]$$

$$\mathcal{L}[\phi + \alpha\Delta\phi] \neq \mathcal{L}[\phi + \alpha\Delta\phi]$$

$$\mathcal{L} \Rightarrow \mathcal{L} + \alpha \partial_\mu J^\mu$$

$$j^\mu = \sum \frac{\partial \mathcal{L}}{\partial(\partial_\mu \phi)} \Delta\phi - J^\mu$$

$$\phi \to \phi + \alpha \Delta\phi \qquad \phi' \to \phi'' \qquad \mathcal{L}' = \mathcal{L} \qquad J^\mu = 0$$

$$\phi = \mathrm{Re}\,\phi + i\,\mathrm{Im}\,\phi$$

$$\mathcal{L} = \partial_\mu \bar\phi \partial^\mu \phi - m^2 \bar\phi \phi = (\phi, \bar\phi) \qquad \boxed{\mathcal{L}^* = \mathcal{L}}$$

$$\phi \to \phi e^{i\alpha}, \; \alpha = \text{const} \simeq \phi + i\alpha\phi \Rightarrow j^\mu = [i\partial^\mu\bar\phi\,\phi - i\partial^\mu\phi\,\bar\phi]$$

$$\bar\phi = \bar\phi e^{-i\alpha} = \bar\phi - i\alpha\bar\phi$$

$$\phi = \phi e^{i\alpha(x)} \Rightarrow S \Rightarrow S + \int d^4x \left[-\partial_\mu(\alpha i\phi)\partial^\mu\bar\phi + \partial_\mu\bar\phi\,\partial^\mu(i\alpha\phi) \right]$$

$$\phi + i\alpha(x)\phi \qquad (\bar\phi - i\alpha\bar\phi)(\phi + i\alpha\phi) = |\phi|^2 + o(\alpha)$$

$$\int dx \left(-i\,\bar\phi\partial^\mu\phi + \partial_\mu\bar\phi\,\phi \right)\partial_\mu\alpha$$

$$= -\int dx\, \alpha \cdot \partial_\mu(j^\mu)$$

$$S(\phi(x^\mu + a^\mu)) \qquad \phi = \phi + a^\nu\partial_\nu\phi + \dots$$

$$S(\phi(x))$$

$$T_{\mu\nu} = j_{\mu\nu} = \frac{\partial\mathcal{L}}{\partial(\partial\phi)}\cdot\partial_\nu\phi - J_{\mu\nu} \qquad \mathcal{L}\eta_{\mu\nu} \qquad \partial^\mu J_\mu$$

$$\mathcal{L} \to \mathcal{L} + a_\mu \partial^\mu \mathcal{L} = \mathcal{L} + a_\nu \partial^\mu \eta$$

$$\partial^\mu (\underbrace{a^\nu \mathcal{L} \eta_{\mu\nu}}_{a_\mu})$$

$$T_{\mu\nu} = \dots - \eta_{\mu\nu}\mathcal{L}$$

$$\mathcal{L} = (\partial_\mu \phi)^2 = (\partial_\mu(\phi + a^\nu \partial_\nu \phi))^2 \qquad (\partial_\mu \phi + \partial_\mu a^\nu \partial_\nu \phi)^2$$

$$= (\partial_\mu \phi)^2 + 2\partial_\mu\phi \cdot a^\nu \partial^\mu \partial_\nu \phi + \dots$$

$$a^\nu (2\partial_\mu \phi (\partial_\mu \partial_\nu \phi))$$

$$\boxed{a^\nu \cdot \partial_\nu (\partial_\mu \phi)^2} = a^\nu \partial_\nu \mathcal{L}, \quad (\partial_\mu\phi)^2 = \mathcal{L}$$