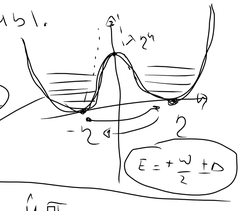
Инстантоны.

$U(x) = \lambda (x^2-\eta^2)^2$

$H = -\frac{1}{2}\partial_x^2 + U(x)$

$S \sim \int_{-\eta}^{\eta} p\,dx \sim \eta \cdot \sqrt{\lambda\eta^2} = \lambda\eta^3 \gg 1$

$\omega^2 = 8\lambda\eta^2$



$$G(x,y,-i\tau) = \langle x| e^{-\hat H \tau} |y\rangle = \sum_n \psi_n^*(x)\psi_n(y) e^{-E_n\tau} \xrightarrow[\tau\to\infty]{} \psi_0(x)\psi_0(y) e^{-E_0\tau}$$

$t = -i\tau$

$$G_E(x,y,\beta) = \int_{x(-\beta/2)=y}^{x(\beta/2)=x} D x(\tau) \exp\left(-\int_{-\beta/2}^{\beta/2} d\tau \underbrace{\left(\frac{1}{2}(\partial_\tau x)^2 + U(x(\tau))\right)}_{\mathcal{L} = T-U}\right)$$

$T + U$

x^4 $\quad \partial_\tau^2$

e^{-S}

$\int dx\, e^{-\frac{1}{2}x^T A x} = \frac{1}{\sqrt{\det \frac{A}{2\pi}}}$

e^f $\quad \frac{\partial f}{\partial x} = 0$

$e^{-S_{d\tau}} \to \frac{\delta S}{\delta x} = 0$

$G_E(\eta,\eta,\beta) = G(\eta,-\eta,\beta) \simeq G_0(\beta) = \sqrt{\frac{\omega}{2\pi \sinh\beta\omega}} \simeq \sqrt{\frac{\omega}{\pi}} e^{-\beta\omega/2}$

$G_E(-\eta,\eta,\beta) = ?$ $\quad \beta\to\infty$

$\partial_\tau^2 x = U'_x$ $\quad E = \frac{1}{2}(\partial_\tau x)^2 - U(x) = 0$

$x_{cl}(\tau) = \eta\, \mathrm{th}\, \frac{\omega(\tau-\tau_0)}{2}$

$$S_0 = \int_{-\infty}^{+\infty} d\tau\, (\partial_\tau x_{cl})^2 = \frac{2}{3}\omega\eta^2 \gg 1$$

$\omega^{-1} \ll \beta$

$$G = \int^0 D[\delta x]\, e^{-S[x]} \; ; \quad x = x_{cl} + \delta x \; ; \quad S(x_{cl}+\delta x) = \underbrace{S(x_{cl})}_{S_0} + S_2(\delta x) + \dots$$

$$S_2 = \int d\tau \left(\frac{1}{2}(\partial_\tau \delta x)^2 + \frac{1}{2}U''(x_{cl})(\delta x)^2\right)$$

$$U''(x_{cl}) = \omega^2\left(1 - \frac{3/2}{\mathrm{ch}^2\frac{\omega\tau}{2}}\right)$$

$$G \approx e^{-S_0}\int D[\delta x]\, e^{-S_2[\delta x] + \dots}$$

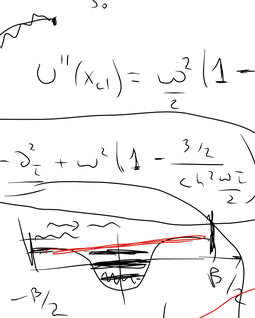
$$\hat L = -\partial_\tau^2 + \omega^2\left(1 - \frac{3/2}{\mathrm{ch}^2\frac{\omega\tau}{2}}\right)$$

$U(x+\delta x) = U(x) + \dots$

$(\partial(x+\delta x))^2 = (\partial x_{cl})^2 + 2\partial x_{cl}\partial_\tau \delta x + (\partial_\tau \delta x)^2$

$$S = \frac{1}{2}\int d\tau\left(-\delta x\, \partial_\tau^2 \delta x + \omega^2 U''\delta x\right)$$

$$\frac{1}{2}\int d\tau\left(\delta x\, \hat L\, \delta x\right)$$



$$\psi_0 = \frac{C}{\mathrm{ch}^2\frac{\omega\tau}{2}}$$

$$\epsilon = \begin{cases} 0 : & \epsilon_0 = 0 \\ 1 : & \epsilon_1 = \frac{3}{4}\omega^2 \end{cases}$$

$E_k = k^2 + \omega^2$

$\delta_k = 2\,\mathrm{arctg}\frac{k}{\omega} + 2\,\mathrm{arctg}\frac{2k}{\omega} = \begin{cases} \to 2\pi \\ \to -2\pi \\ 0 \end{cases}$

$|t| = 1$ $\quad e^{ikx}$ $\quad e^{i(kx+\delta(x))}$

$k_n\beta - \delta(k_n) = \pi n$

$$k_n = \frac{\pi n}{\beta} + \frac{\delta(k_n)}{\beta} \simeq \frac{\pi n}{\beta} + \frac{\delta\left(\frac{\pi n}{\beta}\right)}{\beta} + \dots \quad \beta\to\infty$$

$S_2(\delta x) = \frac{1}{2}\int d\tau\, \delta x \hat L \delta x = \frac{1}{2}\sum_n c_n^2 \lambda_n$

$\int d\tau (\psi_n)^2 = 0$

$\delta x = \sum c_n \psi_n$

$$G = e^{-S}\int \dots$$

$$\frac{G}{G_0} = e^{-S}\frac{\int e^{\int \delta x \hat L \delta x\, d\tau}}{\int e^{\hat L_0}} =$$

$\hat L_0 = -\partial_\tau^2 + \omega^2$

$E_n^{(0)} = p^2 + \omega^2$

$p = \frac{\pi n}{\beta}$

$$= e^{-S}\frac{\prod_n dc_n\, e^{-\frac{1}{2}c_n^2\lambda_n}}{\prod_n dc_n\, e^{-\frac{1}{2}c_n^2\lambda_n^0}} = e^{-S}\frac{\int dc_0\, e^0}{\int dc_0\, e^{-c_0^2(\omega^2+\frac{0\pi^2}{\beta^2})/2}} \cdot \frac{\int dc_1\, e^{-\frac{3}{4}\omega^2 c_1^2}}{\int dc_1\, e^{-(\omega^2+\frac{\pi^2}{\beta^2})c_1^2/2}} \cdot \prod_{i\geq 2}\frac{\int dc_2\, e^{-(k_i^2+\omega^2)c_i^2/2}}{\int dc_2\, e^{-(p_i^2+\omega^2)c_i^2/2}} =$$

$\sqrt{\frac{4}{3}}$ $\quad \sqrt{12}$ $\quad \sqrt{g}$

$$e^{-S_0} \times \left(\frac{\omega}{\sqrt{2\pi}}\int dc_0 \cdot 1\right) \times \sqrt{\frac{\omega^2}{\frac{3}{4}\omega^2}} \cdot \prod_{i\geq 2}\sqrt{\frac{p_i^2+\omega^2}{k_i^2+\omega^2}}$$

$$\frac{1}{g} = e^{+2\ln 3}$$

$$= \exp\left(\frac{1}{2}\sum_{i\geq 2}\ln\frac{p_i^2+\omega^2}{k_i^2+\omega^2}\right)$$

$k_i = p_i + \frac{\delta(p_i)}{\beta}$ $\quad p_i = \frac{\pi n}{\beta}$

$dp = \frac{\pi d i}{\beta} \to \sum di = \int \frac{dp\,\beta}{\pi}$

$$\exp\left(\frac{\beta}{2}\int_0^\infty \frac{dp}{\pi}\ln\frac{p^2+\omega^2}{\left(p+\frac{\delta}{\beta}\right)^2+\omega^2}\right) \xrightarrow{\beta\to\infty}$$

$$\beta\int_0^\infty \frac{dp}{2\pi}\ln\left(1 + \frac{\left(p+\frac{2\pi}{\beta}\right)^2 - \left(p+\frac{\delta}{\beta}\right)^2}{p^2+\omega^2}\right) \simeq \frac{\beta}{2\pi}\int \frac{2p\left(\frac{2\pi}{\beta} - \frac{\delta}{\beta}\right)}{p^2+\omega^2}dp = \int_0^\infty \frac{(2\pi - \delta(p))\,p\,dp}{\pi(p^2+\omega^2)} = 2\frac{\pi\ln 3}{\pi}$$

$$\int_0^\infty dp\left(\frac{1}{1+a^2p^2} + \dots\right)\ln\frac{\omega^2+p^2}{\omega^2}$$

$$+\frac{1}{\pi}\int_0^\infty dp\frac{d\delta(p)}{dp}\ln\left(\frac{\omega^2+p^2}{\omega^2}\right)$$

$e^{-\beta E_0}$ $\quad \delta E = e^{-S}\sqrt{\frac{6\omega S_0}{\pi}}$

$\delta x_0 = c_0\psi_0$

$S(x_{cl}+\delta x_0) = S(x_{cl})$

$S(x_{cl}(t+\delta t)) = S(x_{cl})$

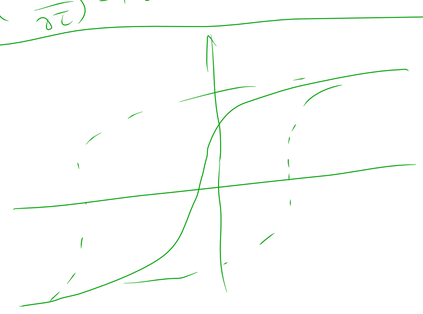
$\frac{\partial x_{cl}}{\partial t}\delta t = c_0\psi_0$

$\eta\, \mathrm{th}\frac{\omega t}{2} \to \boxed{\frac{\eta\,\delta t\,\frac{\omega}{2}}{\mathrm{ch}^2\frac{\omega t}{2}} = c_0\psi_0}$

$\psi_0 = \frac{1}{\sqrt{S_0}}\frac{\partial x_{cl}}{\partial t} \to \boxed{dc_0 = \sqrt{S_0}\,dt}$

$\int d\tau(\psi_0)^2 = 1$

$\to \int d\tau\left(\frac{\partial x_{cl}}{\partial \tau}\right)^2 = S_0$



$$\boxed{\Delta = \frac{2\omega}{\sqrt{\pi}}\sqrt{\omega\eta^2}\, e^{-\frac{2}{3}\omega\eta^2}}$$

$E = \frac{\omega}{2} \pm \Delta$

$$\sum_n \frac{(\Delta\beta)^{2n+1}}{(2n+1)!}$$

$\Delta\beta \gg 1$

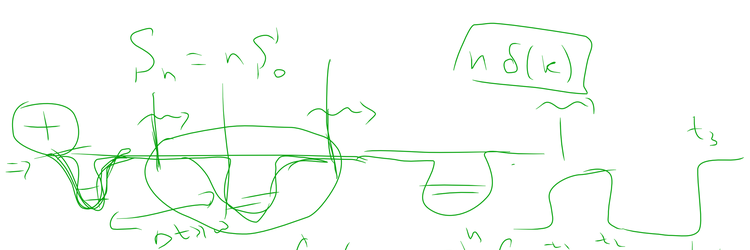
$(\Delta\beta)^n$ $\quad 1/n!$ $\quad n^*$

$(\Delta\beta) \simeq n^* \ll \beta\omega \simeq n \to \boxed{\Delta \ll \omega}$

$\mathrm{th}\frac{\tau\omega}{2}$ $\quad (\Delta\tau = \omega^{-1})$

$S_n = nS_0$

$\boxed{n\,\delta(k)}$



$$\Rightarrow (n\cdot\beta\Delta) = e^{-nS_0}\left(\sqrt{\frac{12S_0\omega}{2\pi}}\right)^n \int_{t_1<t_2<\dots<t_n} dt_1\, dt_2 \dots dt_n = \frac{(\beta)^n}{n!}$$

$e^{+\beta\Delta}$

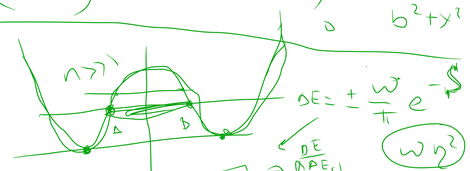
$$\frac{G}{G_0} = \mathrm{sh}(\beta\Delta)$$

$$G_0 \sim \sqrt{\frac{\omega}{\pi}}\, e^{-\omega/2\beta}$$

$\Delta = e^{-S_0}\sqrt{\frac{6\omega S_0}{\pi}}$

$\sin\Delta t \xrightarrow{t=-i\tau} \mathrm{sh}\,\Delta\tau$

$$\int_0^\infty \frac{\ln(a^2+x^2)}{b^2+x^2}dx = \dots$$



$\Delta E = \pm\frac{\omega}{\pi}e^{-S}$ $\quad \omega\eta^2$

$\mathrm{th}\, n e^{-x^2}$ $\quad \sqrt{\frac{\pi}{e}}$ $\quad \frac{\Delta E}{\Delta E_{cl}}$ $\quad n e^{-x^2}$

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