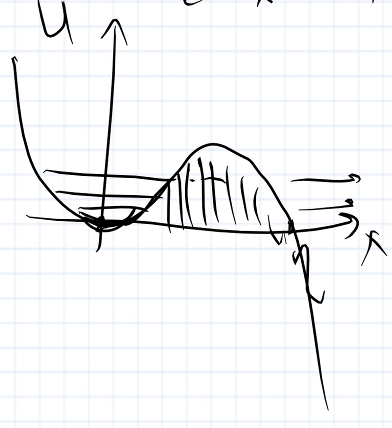
Квантовая механика

Распад метастабильного сост.

$m=1$

$\hat{H} = -\frac{1}{2}\partial_x^2 + \lambda x^2(\eta - x)$



$\omega \approx \sqrt{2\lambda\eta}$

$S \gg 1$

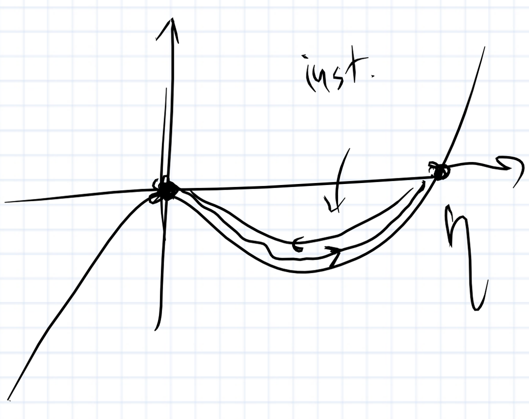
$E = E_n^{(0)} - \frac{i\Gamma_n}{2}$, $\Gamma_n \sim \frac{1}{\tau_n}$ – время жизни.

$E_n^{(0)} \approx \omega(n+\frac{1}{2})$

$E \in (-\infty, \infty)$

$G_E(0,0,\beta) = \langle 0|e^{-\beta\hat{H}}|0\rangle = \sum_n |\psi_n(0)|^2 e^{-\beta E_n} \approx |\psi_0(0)|^2 e^{-\beta E}$

$G_E(0,0,\beta) = \int_{x(-\beta/2)=0}^{x(\beta/2)=0} \mathcal{D}x(t)\, \exp(-S_E)$, $S_E = \int\left[\frac{1}{2}(\partial_\tau x)^2 + U(x)\right]dt$

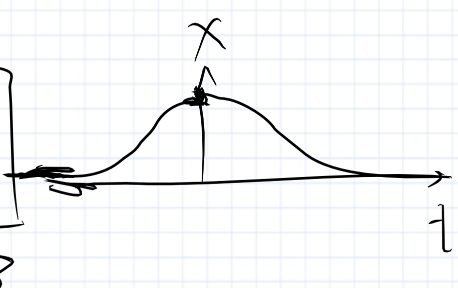


$\partial_\tau^2 x = U'(x) = \lambda x(2\eta - 3x)$

$E = \frac{1}{2}(\partial_\tau x)^2 - U(x) = 0$

$\partial_\tau x = \sqrt{2U(x)}$

$\frac{dx}{\sqrt{2U(x)}} = d\tau \Rightarrow \boxed{x_{cl} = \frac{\eta}{ch^2(\frac{\omega\tau}{2})}}$



$S_{cl} = \int_{-\infty}^{\infty} (\partial_\tau x_{cl})^2 d\tau = \frac{8}{15}\omega\eta^2 \gg 1$

$x(t) = x_{cl}(t) + z(t)$

$S[x(t)] - S[x_{cl}(t)] = \frac{1}{2}\int z(\tau)\{-\partial_\tau^2 + U''(x_{cl}(\tau))\} z(\tau)\, d\tau$

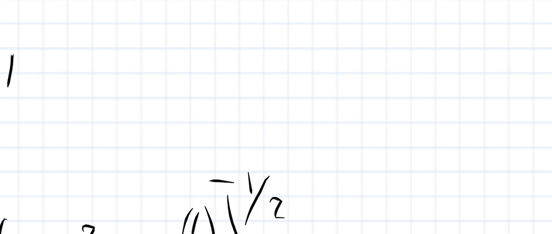
$\hat{L} = -\partial_\tau^2 + \omega^2\left(1 - \frac{3}{ch^2\frac{\omega\tau}{2}}\right)$

$\hat{L}\psi_n(x) = \epsilon_n \psi_n(x)$

Нулевая мода: $\psi_1(\tau) = \frac{1}{\sqrt{S_0}}\frac{\partial x_{cl}}{\partial\tau}$

$\epsilon_1 = 0$

$\epsilon_0 < 0$



← 1 возб. сост.

$z(\tau) = \sum_n c_n z_n(\tau)$

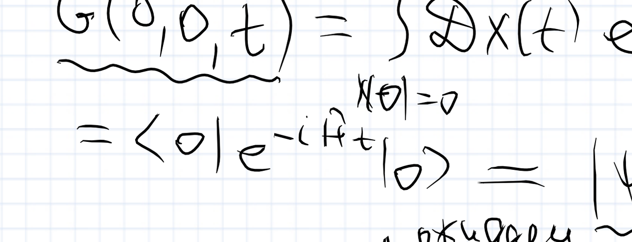
$\int dc_0 \exp(-\frac{1}{2}\epsilon_0 c_0^2) = \sqrt{\frac{2\pi}{\epsilon_0}} = \pm i\sqrt{\frac{2\pi}{|\epsilon_0|}}$ ← расх-ся!

отриц. мода

$\frac{G_E(0,0,\beta)}{G_0(0,0,\beta)} = \pm i\, e^{-S_0} \cdot \sqrt{\frac{S_0}{2\pi}}\,\omega\beta \times \left(\frac{\det'(-\partial_\tau^2 + U'')}{\det'(-\partial_\tau^2 + \omega^2)}\right)^{-1/2} \equiv \pm i\frac{\Gamma_0}{2}$

inst.; нулевая мода; $-\frac{1}{60}$

Инстантонный газ



$\sum_{N=0}^{\infty} \int_{\tau_1 < \ldots < \tau_N} d\tau_1 \ldots d\tau_N \cdot \left(\pm i\frac{\Gamma_0}{2}\right)^N = \exp\left(\pm i\frac{\Gamma_0\beta}{2}\right)$, $\int d\tau_1 \ldots d\tau_N = \frac{\beta^N}{N!}$

$\Rightarrow \Delta E = \pm i\frac{\Gamma_0}{2}$

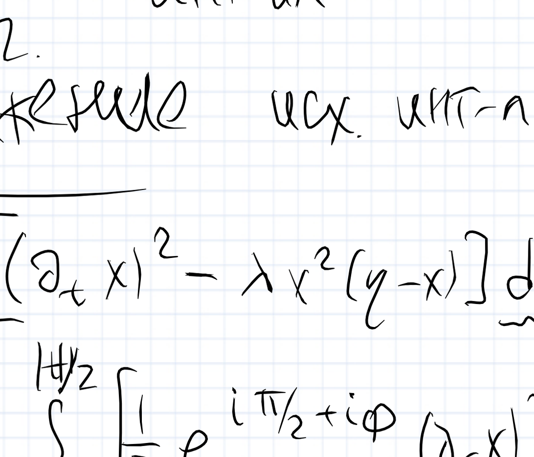
$\Gamma_0 = \omega\sqrt{\frac{30 S_0}{\pi}}e^{-S_0}$ (истинная Г в 2 раза меньше!)

$G(0,0,t) = \int_{x(0)=0}^{x(t)=0} \mathcal{D}x(t)\, \exp(iS)$, $S[x(t)] = \int\left[\frac{1}{2}\dot{x}^2 - U(x)\right]dt$

$= \langle 0|e^{-i\hat{H}t}|0\rangle = |\psi_0(0)|^2 e^{-iE_0 t} e^{-\Gamma_0 t/2}$ ← ожидаем

$t = -i\beta$

$|\psi_0|^2 e^{-\beta E_0} e^{+i\Gamma_0\beta/2}$



Выше: аналитич. продолжение исх. инт-ла

$iS = i\int_{-t/2}^{t/2}\left[(\partial_t x)^2 - \lambda x^2(\eta - x)\right]dt =$ $t \mapsto |t|e^{-i\varphi}$, $\tau = |t| = te^{+i\varphi}$

$iS[t,\lambda,\eta] = \int_{-|t|/2}^{|t|/2}\left[\frac{1}{2}e^{i\pi/2 + i\varphi}(\partial_\tau x)^2 - \lambda e^{i\frac{\pi}{2} - i\varphi}x^2(\eta - x)\right]$

($\lambda e^{i\frac{\pi}{2}-i\varphi}$ – чисто мнимой)

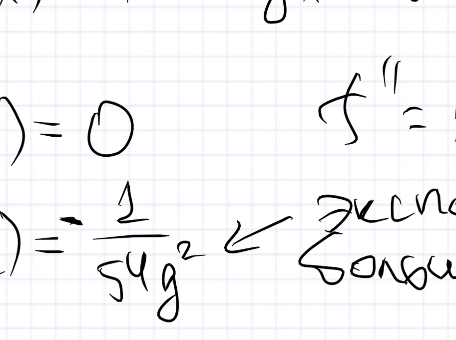
$G(\lambda, t) = G(\lambda e^{i\varphi}, te^{-i\varphi}) = G_E(i\lambda, \beta)$, $\beta = |t|$

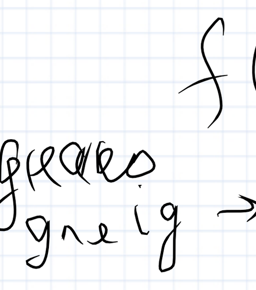
$\varphi \mapsto \frac{\pi}{2}$

Модельный интеграл

$I(g) = \int_{-\infty}^{+\infty}\frac{dx}{\sqrt{2\pi}}\exp\left(-\frac{x^2}{2} + igx^3\right)$, $-f(x)$, $g \ll 1$

$I(ig)$?





$f(x) = \frac{x^2}{2} - igx^3$

$f'(x) = x - 3igx^2 = 0 \Rightarrow \left[\begin{array}{l} x = 0 \\ x_1 = -i/3g \end{array}\right.$

$f(x_1) = 0$ $\quad f'' = 1 - 6igx = \left[\begin{array}{l} 1 \\ -1 \end{array}\right.$

$f(x_2) = -\frac{1}{54g^2}$ ← экспоненц. большой вклад конца.

$f(\tilde{x}_1) \approx -\frac{1}{54g^2} - \frac{(x - x_1)^2}{2}$

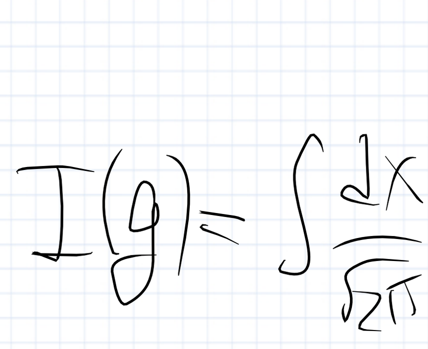
Однако для $ig \to \frac{1}{54|g|^2}$ $\quad \int\frac{dx}{\sqrt{2\pi}}e^{(x-x_1)^2/2}$

Аналит. продолж.:

$I(ge^{-i\varphi}) = \int_C dx\, e^{-\frac{x^2}{2} + ige^{-i\varphi}x^3} =$ $\quad x' = xe^{+i\varphi/3}$

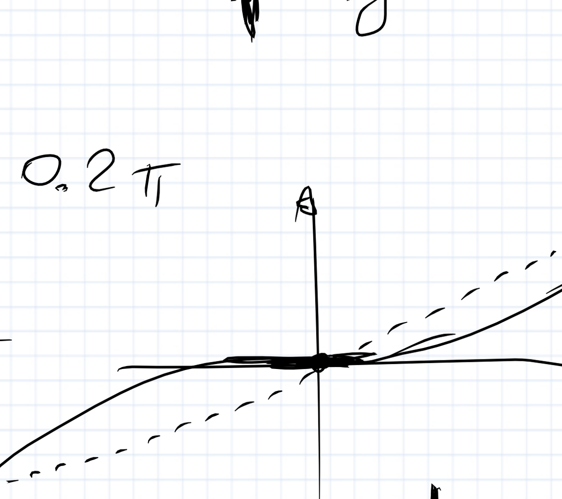
$= \int_{-\infty}^{\infty} dx\, \exp\left(-e^{2i\varphi/3}\frac{x^2}{2} + igx^3\right)$

$C = \{-\infty \cdot e^{+i\varphi/3}; +\infty e^{+i\varphi/3}\}$



1) $\varphi = 0$

$I(g) \approx 1$



$I(g) = \int\frac{dx}{\sqrt{2\pi}}e^{-x^2/2 + igx^3}$

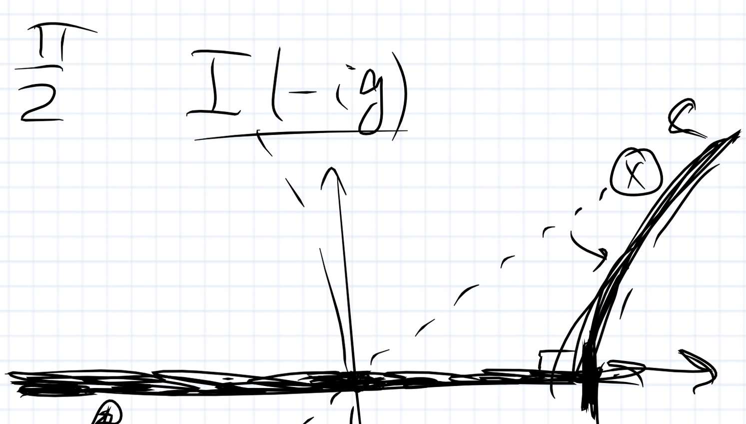
"как бы взял" $g \to -ig$

$I(-ig) = \int_C\frac{dx}{\sqrt{2\pi}}e^{-x^2/2 + gx^3}$

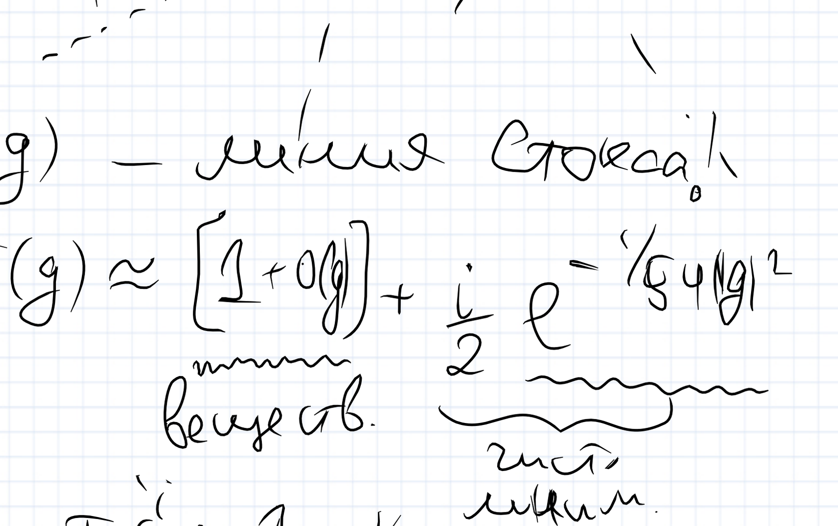
$\text{Im}\, I(-ig) = \frac{1}{2}e^{-\frac{1}{54|g|^2}}$

2) $\varphi = -0.2\pi$

$I(g) \approx 1$



3) $\varphi \mapsto \frac{\pi}{2}$ $\quad I(-ig)$



$I(-ig)$ – линия Стокса!

$I(g) \approx [1 + O(g)] + \frac{i}{2}e^{-1/54|g|^2}$

(веществ.; чисто мним.)

$\text{Im}\, I(ig) = \frac{1}{2}e^{-1/54|g|^2}$

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