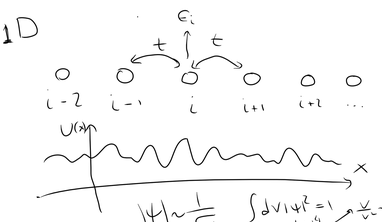
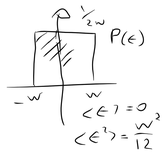
Андерсоновская локализация.

1D





$\langle\epsilon\rangle = 0$

$\langle\epsilon^2\rangle = \frac{w^2}{12}$

$t \gg w$

$\psi \sim e^{ikx}$

$|\psi| \sim \frac{1}{\sqrt{V}}$ $\quad \int dV |\psi|^2 = 1$ $\quad \frac{V}{v_0} - \frac{t}{v_0} \to 0$

$I_n = \int dV |\psi|^n \to C$

$$\hat{H} = \sum_i \left(\epsilon_i \hat{a}_i^+ \hat{a}_i - t\,\hat{a}_{i+1}^+ \hat{a}_i + h.c.\right) \to \begin{pmatrix} \epsilon_i & -t & \\ -t & \epsilon_{i+1} & -t \\ & -t & \end{pmatrix}$$

$\epsilon_i \to 0 \qquad E(k) = -2t\cos k$

$(t=1)$

$\hat{H}|\psi\rangle = E|\psi\rangle \to E\psi_i = \epsilon_i \psi_i - (\psi_{i+1} + \psi_{i-1})$

*) $2 \to 1$; $X_i = \begin{pmatrix}\psi_i \\ \psi_{i-1}\end{pmatrix} \to X_{i+1} = \hat{m}_i X_i$

$\hat{m}_i = \begin{pmatrix} \epsilon_i - E & -1 \\ 1 & 0 \end{pmatrix}$

$\begin{pmatrix}\psi_{i+1} \\ \psi_i\end{pmatrix} = \begin{pmatrix} & \\ 1 & 0 \end{pmatrix}\begin{pmatrix}\psi_i \\ \psi_{i-1}\end{pmatrix}$

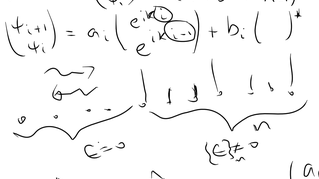
(**) $\begin{pmatrix}\psi_{i+1} \\ \psi_i\end{pmatrix} = a_i \begin{pmatrix} e^{ik} \\ e^{ik(i-1)} \end{pmatrix} + b_i \begin{pmatrix} \\ \end{pmatrix}^*$

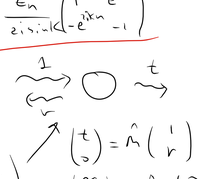
$$X_n = \underbrace{\begin{pmatrix} e^{ik_n} & e^{-ik_n} \\ e^{ik_n(n-1)} & e^{-ik_n(n-1)} \end{pmatrix}}_{R_n^{fact}} \begin{pmatrix} a_n \\ b_n \end{pmatrix}$$

$X_{n+1} = R_{n+1}\begin{pmatrix} a \\ b \end{pmatrix}_{n+1} = m_n R_n \begin{pmatrix} a \\ b \end{pmatrix}_n$ $\quad \det R_n = e^{ik} - e^{-ik}$

$\Rightarrow \tilde{m}_n = R_{n+1}^{-1}\, \hat{m}_n\, R_n$ $\quad \det \tilde{m}_n = 1$

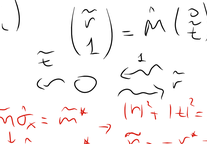
$$\tilde{m}_n = \hat{\mathbb{1}} + \frac{\epsilon_n}{2i\sin k}\begin{pmatrix} 1 & e^{-2ik_n} \\ -e^{2ik_n} & -1 \end{pmatrix}$$





$\begin{pmatrix} t \\ 0 \end{pmatrix} = \hat{M}\begin{pmatrix} 1 \\ r \end{pmatrix}$

$\begin{pmatrix} \tilde{r} \\ 1 \end{pmatrix} = \hat{M}\begin{pmatrix} 0 \\ \tilde{t} \end{pmatrix}$



$\hat{M} = \prod_n \tilde{\hat{m}}_n$

$\begin{pmatrix} a_R \\ b_R \end{pmatrix} = \hat{M}\begin{pmatrix} a_L \\ b_L \end{pmatrix}$

$$\hat{M} = \begin{bmatrix} t - \frac{r\tilde{r}}{\tilde{t}} & \frac{r}{\tilde{t}} \\ -\frac{\tilde{r}}{\tilde{t}} & \frac{1}{\tilde{t}} \end{bmatrix} \qquad t = \tilde{t}$$

$\det \hat{M} = 1$

$$\begin{cases} \hat{\sigma}_x \tilde{m} \hat{\sigma}_x = \tilde{m}^* \\ \hat{\sigma}_x M \hat{\sigma}_x = M^* \end{cases} \to |r|^2 + |t|^2 = 1, \quad \tilde{r} = -r^* \frac{t}{t^*}$$

$\eta_n - ?$

$\hat{\mathcal{M}} = \hat{M}\hat{M}^+$

$\sigma_0 + \sigma_x + \sigma_y + \sigma_z$

$$\hat{M}_n = \begin{bmatrix} \mathrm{ch}\,\eta_n & e^{i\varphi_n}\mathrm{sh}\,\eta_n \\ e^{-i\varphi_n}\mathrm{sh}\,\eta_n & \mathrm{ch}\,\eta_n \end{bmatrix}$$

$|t|^2 = \frac{1}{\mathrm{ch}^2\eta_n}$

$\mathcal{M}_n + \mathbb{1} \to \mathcal{M}_{n+1}$

$$\mathcal{M}_{n+1} - \mathcal{M}_n = M_{n+1} M_{n+1}^+ - M_n M_n^+ = -\frac{\epsilon_n}{\sin k}\begin{bmatrix} \mathrm{sh}\,\eta_n \sin(2k_n + \varphi_n) & i\,\mathrm{ch}\,\eta_n e^{-2ik_n} + i\,\mathrm{sh}\,\eta_n e^{i\varphi_n} \\ -i\,\mathrm{ch}\,\eta_n e^{2ik_n} - i\,\mathrm{sh}\,\eta_n e^{-i\varphi_n} & -\mathrm{sh}\,\eta_n \sin(2k_n + \varphi_n) \end{bmatrix} + \frac{\epsilon_n^2}{2\sin^2 k}\left[\mathrm{ch}\,\eta_n + \mathrm{sh}\,\eta_n \sin(2k_n + \varphi_n)\right]\begin{bmatrix} 1 & -e^{-2ik_n} \\ -e^{2ik_n} & 1 \end{bmatrix}$$

$\tilde{M}_{n+1}\mathcal{M}_n\tilde{M}_{n+1}^+ - \mathcal{M}_n =$

$$\begin{bmatrix} \mathrm{ch}\,\eta_{n+1} - \mathrm{ch}\,\eta_n & a - ib \\ a + ib & \dots \end{bmatrix}$$

$\mathrm{sh}\,\eta_n(\eta_{n+1} - \eta_n) = (\# \frac{1}{\dots} - \dots)$

$\varphi_n \to \Phi_n - f_{c,s} + 2k_n$

$$\eta_{n+1} = \eta_n - \frac{\epsilon_n \cos\Phi_n}{\sin k}\Phi_n + \frac{\epsilon_n^2 \cos\Phi_n}{2\sin^2 k}\left(\mathrm{ch}\,\eta_n \cos\Phi_n + \dots\right) + 2k$$

$\Phi_n = \varphi_n + 2k_n$

$\frac{k}{\pi} \neq \frac{s}{n}$

$\Phi_{n+1} = \sin^{-1} e = \Phi_n + \epsilon_n \# + \epsilon_n^2 \# \quad (***) \quad \langle\ \rangle_\Phi \to \int \frac{d\Phi}{2\pi}$

$\eta_{n+1} = \{\epsilon_n^2, \Phi_n\}$

$f(x)$

$$P_n(\eta_n) \Big|_{\underbrace{\dots}_{n}} \Rightarrow P_{n+1}(\eta) = \iiint \delta\left(\eta_n - \frac{\epsilon_n \cos\Phi_n}{\sin k} + \epsilon_n^2 F(\Phi_n, \eta_n) - \eta\right) P_n(\eta_n)\, d\eta_n \times P(\epsilon)\, d\epsilon_n \cdot \frac{d\Phi_n}{2\pi}$$

$\int \delta(f(x))\,dx = \frac{1}{|f'(x)|}$

$$\eta_n \simeq \eta + \frac{\epsilon_n}{\sin k}\sin\Phi_n - \epsilon_n^2 F(\Phi_n, \eta_n) \leftarrow x_0$$

$$= \iint P(\epsilon)\, d\epsilon_n \frac{d\Phi}{2\pi}\, P_n\left(\eta + \frac{\epsilon_n}{\sin k}\sin\Phi_n - \epsilon_n^2 F(\Phi, \eta)\right) \frac{1}{1 + \epsilon_n^2 F_\eta'(\eta, \Phi)}$$

$\langle\epsilon\rangle = 0$

$\frac{\partial \arg\delta(\ )}{\partial \eta_n} = 1 + \epsilon_n^2 F_\eta'$

$$= \int P(\epsilon)\, d\epsilon \frac{d\Phi}{2\pi}\left(P_n(\eta) + \frac{\partial}{\partial\eta}P_n(\eta)\left[\frac{\epsilon_n \cos\Phi}{\sin k} - \epsilon_n^2 F\right] + \frac{\partial^2}{\partial\eta^2}P_n(\eta)\left[\frac{\epsilon_n^2}{\sin^2 k}\cos^2\Phi_n\right]\right)\left(1 - \epsilon^2 F_\eta'\right)$$

$\langle\epsilon^2\rangle$ $\quad \langle\epsilon^2\rangle$ $\quad \langle\epsilon^2\rangle$

$$= P_n(\eta)\left(1 - \langle\epsilon^2\rangle\langle F_\eta'\rangle_\Phi\right) + \frac{\partial P}{\partial\eta}\cdot\langle\epsilon^2\rangle\langle F\rangle_\Phi + \frac{\partial^2 P}{\partial\eta^2}\cdot\frac{\langle\epsilon^2\rangle}{2\sin^2 k}\langle\cos^2\Phi_n\rangle$$

$\langle\cos^2\Phi_n\rangle = 1/2$

$$\langle F\rangle_\Phi = \left\langle \cos\Phi \frac{\left(\sqrt{\mathrm{ch}\,\eta}\,\cos\Phi\right)}{2\sin^2 k}\right\rangle = \frac{1}{4\sin^2 k}\,\mathrm{cth}\,\eta$$

$$\langle F_\eta'\rangle_\Phi = \frac{1}{4\sin^2 k}\frac{\partial}{\partial\eta}(\mathrm{cth}\,\eta)$$

$\frac{2}{L_{loc}} : L_{loc} = \frac{8\sin^2 k}{\langle\epsilon^2\rangle} = \frac{96\sin^2 k}{w^2}$

$$P_{n+1}(\eta) - P_n(\eta) \simeq \frac{\partial P}{\partial n} = -\frac{\partial P}{\partial\eta}\frac{\langle\epsilon^2\rangle}{4\sin^2 k}\mathrm{cth}\,\eta - \frac{\langle\epsilon^2\rangle}{4\sin^2 k}P(\partial_\eta \mathrm{cth}\,\eta) + \frac{\partial^2 P}{\partial\eta^2}\frac{\langle\epsilon^2\rangle}{4\sin^2 k}$$

$$\frac{\partial P}{\partial n} = \frac{2}{L_{loc}}\frac{\partial}{\partial\eta}\left[\mathrm{sh}\,\eta\frac{\partial}{\partial\eta}\frac{P}{\mathrm{sh}\,\eta}\right] \leftarrow \int d\eta$$

$\boxed{\int P\, d\eta = 1}$

$\partial_n \int P\, d\eta = 0$

$P(n=0) = \delta(\eta)$

$\eta = \frac{2n}{L_{loc}}$

$|t|^2 = \frac{1}{\mathrm{ch}^2\eta_n} = 1$

$\frac{\partial P}{\partial n} = 0 \to P = \mathrm{sh}\,\eta$

$\eta \to \infty \quad \mathrm{sh} \sim e^\eta/2$

$\eta \gg 1$

$$\frac{\partial P}{\partial n} = \frac{2}{L_{loc}}\left[\frac{\partial^2 P}{\partial\eta^2} - \frac{\partial P}{\partial\eta}\right]$$

$$P_n(\eta) = \sqrt{\frac{L_{loc}}{8\pi n}}\exp\left[-\frac{(\eta - 2n/L_{loc})^2}{8n/L_{loc}}\right]$$

$2n/L_{loc}$

$\eta \simeq 1$

$w^2 \to 0$

$T_{typ} \sim \frac{1}{\mathrm{ch}^2\eta_n} \sim e^{-\eta} \sim \# e^{-2n/L_{loc}}$

$\ln\langle\exp(-T)\rangle$

$$\langle T\rangle = \int_0^\infty e^{-\eta} P_n(\eta)\, d\eta \simeq \# e^{-n/2L_{loc}}$$

- $L_{loc} \sim \frac{1}{w^2}$ (1D)
- (2D)
- $L_{loc} \sim e^{\frac{1}{w^2}}$ $\quad \langle w^2\rangle < w_c^2$ (3D)
- $L \to \infty$
- $L \to L$ $\quad \langle w^2\rangle > w_c^2$

DMPK

$$\boxed{\frac{\partial P}{\partial n} = \frac{2}{e}\partial_\eta\, \mathrm{sh}\,\eta \frac{\partial}{\partial\eta}\frac{P}{\mathrm{sh}\,\eta}}$$

Created with IDroo.com