Модель Калдейры-Леггета

$$\hat H = \hat H_s + \hat H_{env} + \hat H_{int}$$

$$\hat H_s = \frac{\hat p^2}{2m} + \hat V(\hat x)$$

$$\hat H_{env} = \sum_n \left(\frac{\hat p_n^2}{2m_n} + \frac{m_n \omega_n^2 \hat x_n^2}{2}\right)$$

$$\hat H_{int} = \sum_n \left(-c_n \hat x_n \cdot \hat X + \frac{c_n^2}{2m\omega_n^2}\hat X^2\right)$$

$$\to \frac{1}{2} m\omega_n^2 \left(x_n - \frac{c_n}{m\omega_n^2} X\right)^2$$

Кинетическое ур-ние

$\hat\rho(x, x', t)$

Представление взаимод-я по отн. к $\hat H_{env}$

$\hat x_n(t)$

$$\frac{d\hat\rho_s}{dt} = i[\hat\rho_s(t), \hat H_s] - \int_0^\infty d\tau\, Tr_e\left([[\hat\rho_s(t)\hat\rho_e, \hat H_{int}(t-\tau)], \hat H_{int}(t)]\right)$$

$$\hat H_{int} = -\hat X\,\hat\Phi(t)$$

$$\hat\Phi_H = \sum_n c_n \hat x_n(t)$$

Линейные по отн. к (y_n, p_n)

$$\begin{cases} \frac{d\hat X}{dt} = \frac{\hat p}{m} \\ \frac{d\hat p}{dt} = -\frac{\partial V}{\partial X} + \sum_n c_n \hat y_n \end{cases} \qquad \begin{cases} \frac{d\hat y_n}{dt} = \frac{\hat p_n}{m_n} - \frac{c_n}{m\omega_n^2}\frac{d\hat X}{dt} \\ \frac{d\hat p_n}{dt} = -m_n\omega_n^2 \hat y_n \end{cases}$$

$$y_n \equiv x_n - \frac{c_n}{m\omega_n^2}\hat X$$

$$\Downarrow$$

$$\frac{d^2\hat y_n}{dt^2} + \omega_n^2 \hat y_n = -\frac{c_n}{m\omega_n^2}\frac{d^2\hat X}{dt^2}$$

Ф-ция Грина: $G_n(t) = \frac{\sin(\omega_n t)}{\omega_n}\theta(t)$

$$G(0) = 0$$
$$G'_+(0) = 1$$

$$\hat y_n(t) = \hat y_n(0)\cdot \dot G_n(t) + \left(\frac{\hat p_n(0)}{m_n} - \frac{c_n}{m\omega_n^2}\dot{\hat X}(0)\right) - \frac{c_n}{m\omega_n^2}\int_0^t G_n(t-\tau)\cdot\frac{d^2\hat X(\tau)}{d\tau^2}\,d\tau =$$

$$= \hat y_n(0)\dot G_n(t) + \frac{\hat p_n(0)}{m_n}G_n(t) - \frac{c_n}{m\omega_n^2}\int_0^t \dot G_n(t-\tau)\frac{d\hat X}{d\tau}(\tau)\,d\tau$$

$$M\frac{d^2\hat X}{dt^2} + \frac{\partial V}{\partial X} - \sum_n c_n \hat y_n(t) = 0$$

$$\Rightarrow \boxed{M\frac{d^2\hat X}{dt^2} + \int_0^t d\tau\,\gamma(t-\tau)\frac{d\hat X(\tau)}{d\tau} + \frac{\partial\hat V}{\partial X} = \hat\xi(t)}$$

$$\boxed{\gamma(t) = \sum_n \frac{c_n^2}{m_n\omega_n^2}\dot G_n(t)}$$

$$\boxed{\hat\xi(t) = \sum_n c_n\left(\hat y_n(0)\dot G_n(t) + \frac{\hat p_n(0)}{m_n}G_n(t)\right)}$$

Спектральная ф-ция шума

$$J(\omega) = \pi\sum_n \frac{c_n^2}{2m\omega_n}\delta(\omega-\omega_n)$$

$$\boxed{\gamma(t) = \frac{2}{\pi}\int_0^\infty \frac{d\omega}{\omega}J(\omega)\cos(\omega t)} \quad \hat H_{int}$$

$$\hat\rho_e(0) = e^{-H_e/T}$$

$$\langle\hat\xi(t)\rangle_e = 0$$

$$\langle\hat\xi(t)\hat\xi(t')\rangle = f(t-t')$$

$$\langle\hat y_n^2(0)\rangle = \frac{1}{2m_n\omega_n}\,cth\frac{\beta\omega_n}{2} \quad \to \quad H_e = \sum_n\left(\frac{p_n^2}{2m_n} + \frac{m_n\omega_n^2 y_n^2}{2}\right) = \sum \hbar\omega_n\left(\hat a_n^+\hat a_n + 1/2\right)$$

$$\langle\hat p_n^2(0)\rangle = \frac{m_n\omega_n}{2}\,cth\frac{\beta\omega_n}{2}$$

$$\rho_{e,k} \propto \exp\left(-\frac{\hbar\omega_n}{T}(k+1/2)\right)$$

$$\left\langle\frac{m\omega_n^2\hat y_n^2(0)}{2}\right\rangle = \left\langle\frac{\hat p_n^2}{2m}\right\rangle = \frac{1}{2}\langle\hat H\rangle$$

$$\Leftrightarrow \langle k\rangle \equiv \langle\hat a^+\hat a\rangle = n_B(\omega) = \frac{1}{\exp\left(\frac{\hbar\omega}{T}\right)-1}$$

$$\langle\hat H\rangle = \frac{1}{2}\hbar\omega\, cth\left(\frac{\hbar\omega}{2T}\right) = \begin{cases}\frac{\hbar\omega}{2}, & T\to 0\\ T, & T\to\infty\end{cases}$$

$$n_B + 1/2 = \frac{1}{2}cth(\dots)$$

$$\boxed{f(t) = \frac{1}{\pi}\int_0^\infty d\omega\, J(\omega)\,cth\frac{\beta\omega}{2}\cos\omega t}$$

$$f(\omega) = \gamma(\omega)\,\frac{\omega}{2}\,cth\frac{\beta\omega}{2} \leftarrow \text{ФДТ}$$

Омическая баня: $\omega_c\to\infty$

$$J(\omega) = \eta\cdot\omega\;\theta(\omega_c-\omega)$$

$$\gamma(t) = 2\eta\,\delta(t)$$

(регуляризация: $\int_0^t d\tau\,\delta(t-\tau) = 1/2$)

$$\gamma(t)\equiv\gamma(-t)$$

$$\Rightarrow M\frac{d^2\hat X}{dt^2} + \eta\frac{d\hat X}{dt} + \frac{\partial V}{\partial X} = \hat\xi(t)$$

$$cth\frac{\beta\omega}{2} \approx \frac{2}{\beta\omega}$$

$$\langle\hat\xi(t)\hat\xi(t')\rangle = T\gamma(t-t')$$